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Procedural and conceptual knowledge of expert and novice students for the solving of a basic problem in chemistry

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Procedural and conceptual knowledge for solving a basic quantitative problem in chemistry by expert and novice secondary school students is reported. Experts use a known qualitative procedure with a working forwards strategy to obtain a numerical solution. Novices attempt a means-ends analysis strategy which is often unsuccessful, so switch to a simple formula-driven working forwards strategy to obtain a numerical solution, the qualitative procedure being either omitted or only partially formed. A gradual shift in strategies and representations used as expertise increases was observed. Differences with findings for problem solving in physics were also found. Experts' conceptual knowledge is accurate and linked to underlying procedural knowledge, whereas novices have misconceptions and a poor understanding of formulae. Conceptual understanding, use of a qualitative procedure, and the type of strategy used, are major differences between experts and novices. Instructional techniques are suggested in these areas to enhance problem solving and teaching.

Introduction

Instruction in science is generally aimed at achieving two goals: the acquisition of a body of organized knowledge in a particular domain and the ability to solve problems in that domain. Much of the problem solving is quantitative, involving formulae and the application of mathematics, and is a source of great difficulty for many students. A major approach for investigating problem solving comes from information-processing psychology (Newell and Simon 1972), the goals of which are to produce knowledge about how individuals think, the mechanisms of their problem solving, the causes of errors, differences between skilled and less skilled performance, and from a teaching perspective, the hope of improving instruction. These goals relate closely to the aims of science education and the approach has been used to obtain explicit models of human problem solving in the domain of physics (Reif 1981, Larkin 1983).

Two basic mental processes are involved in problem solving according to the information-processing approach. One is the construction of representations of the problem based on a conceptual understanding of information given in the problem statement. The second process involves the use of a strategy to guide the search for a solution procedure from the initial state of the problem (the information and data given) to the goal state (the required answer). Other factors influencing problem solving such as memory, cognitive load and task perception are often included in the investigations (Kempa and Nicholls 1983).

Problem-solving strategies

Two strategies have been observed for the solving of problems in a number of subjects including physics and mathematics. These are the 'working forwards strategy' and the 'means-ends analysis strategy' (e.g. Ayres 1993, Larkin 1983, Owen and Sweller 1985, Sweller 1988). With working forwards, the solver begins with the current information in the problem statement and works forwards performing operations to transform it until the goal is reached. In studies by Larkin (1983) of problem solving in physics, expert problem solvers (university professors) seemed to use a working forwards strategy exclusively when solving problems. Working forwards is associated with previous experience in the type of problem being solved and as Kramers-Pals, Lambrechts and Wolff (1983) have pointed out, is an efficient strategy as it saves time because the problem is familiar and the solver knows the procedure for obtaining the answer.

Means-ends analysis is a form of backward reasoning and involves (a) identifying the goal statement, (b) finding differences between the goal and the current information, (c) finding an operation that will reduce this difference (such as using a formula or equation), (d) attempting to carry out this operation, and if this is not possible then (e) repeating steps (b) to (d) recursively with a series of sub-goals until a solution path is found. The procedure created is held in working memory in the reverse order to that which will be used to obtain the written solution. Means-ends analysis is associated with novice problem solving and has also been reported in physics and mathematics with problem solvers ranging from primary school pupils to university students (e.g. Larkin 1983, Sweller 1988, Sweller *et al.* 1983).

Human problem solvers also switch strategies depending on experience with problems. For example, Sweller *et al.* (1983) found a switch from a means-ends strategy to working forwards for the solving of basic kinematics problems in the course of development from primary school to university level as experience with the problems increased. In the physics studies of Larkin (1983), novices (undergraduate students) successfully used a means-ends analysis for problems perceived as easy but for problems perceived as difficult, some novices would begin a means-ends analysis but then switch to a working forwards strategy in an attempt to derive a solution.

Problem representation

How problems are represented features largely in problem-solving research. Studies in physics have shown that the representation of a problem changes while it is being solved and that these changes are qualitatively different for experts and novices (e.g. Chi *et al.* 1981, Coleman and Shore 1991, Larkin 1983). Initially, problem solvers concentrate on some of the key words in the written description of the problem (Chi *et al.* 1981, Larkin 1983). This information is closely tied to real, familiar objects such as pulleys, moving blocks and springs, and forms an important part of the initial representation of problems for both expert and novice problem solvers (e.g. Larkin 1983, Slotta *et al.* 1995). Experts link this initial representation to laws and principles of formal physics to build up a *qualitative* procedure for solving the problem. The qualitative representation is then used to formulate a mathematical representation by guiding the selection of appropriate formulae to obtain a numerical solution. Novices, in contrast, focus on the superficial aspects of the initial representation which enables the behaviour of objects in a real situation to be simulated but which provides little guidance in selecting physics principles for application. Omitting the qualitative thinking, novices construct a mathematical representation by focusing on formulae and equations in order to obtain a numerical solution.

Problem solving and conceptual knowledge

Science subjects, such as physics and chemistry, contain three levels of knowledge, namely, the macroscopic, the microscopic and the symbolic (Johnstone 1991). The macroscopic level is a concrete level corresponding to observable objects, their properties and the terms used to describe them. The microscopic level involves the concepts, theories and principles needed to explain what is observed at the macroscopic level. The symbolic level deals with formulae and mathematical calculations. Scientists and science teachers operate across all three levels of thought quite easily and switch from one mode of thinking to another without effort. Past research indicates that students have great difficulty with the microscopic level and develop many scientific misconceptions (e.g. Garnett *et al.* 1995, Nakhleh 1992). This level, of course, is outside their experience and can only be made accessible through the use of concrete models, analogies and graphics (Gabel 1986, Johnstone 1991). Misconceptions in volumetric analysis, the topic investigated in this study, have been found for the mole and molarity (e.g. Duncan and Johnstone 1973, Novick and Menis 1976, Schmidt 1984, Staver and Lumpe 1995), the concept of volume (Enochs and Gabel 1984), the balancing of chemical equations (Nurrenbern and Pickering 1987, Yarroch 1985) and the particulate nature of matter (Griffiths and Preston 1992, Novick and Nussbaum 1978).

In spite of conceptual difficulties, many students are still able to solve quantitative problems in science correctly (e.g. Gabel Sherwood and Enochs 1984, Stewart 1985). This is done by relying on algorithms, especially for basic or routine problems (Gabel and Bunce 1994). The use of algorithms is not, as might be expected, limited to less able problem solvers. Anamuah-Mensah (1986), for example, found that students of *all* achievement levels used algorithmic approaches for solving titration problems in volumetric analysis at secondary school level. Reasons put forward to account for the dependence on algorithms are that teachers and general chemistry courses frequently emphasize the application of algorithms to solve routine problems (Nurrenbern and Pickering 1987) and that problems met in textbooks include procedures which can be used algorithmically (Bodner 1987).

Method

The present study

This present study investigates quantitative problem solving in an area of secondary school chemistry where students have traditionally experienced a lot of difficulties, i.e. volumetric analysis. The major purpose of the study is to ascertain and compare the procedural knowledge (in terms of strategies used and problem representation) and conceptual knowledge of students classified as 'experts' and 'novices' for one problem in this topic. Comparisons with physics problem solving is made and, based on the findings, suggestions for the enhancement of problem solving and teaching practice are given.

A description of the problem type

In contrast to studies which investigate the solving of relatively complex problems, the problem chosen for investigation is a basic one being one of the first met by students when studying volumetric analysis. Basic problems in volumetric analysis include calculating numbers of moles of a solute, concentration of a solution, changes in concentration when a solution is diluted and concentration of an acid or alkali in a titration. The problem chosen involves calculating the concentration of a solution. The problem is:

> A solution contains 1.1 g of sodium nitrate in 250 cm^3 of solution. What is the concentration of the solution? [Relative atomic masses: Na 23, N 14, O 16]

The standard procedure for solving this problem has two main steps: (a) The calculation of the number of moles of solute (sodium nitrate) using the formula moles = mass/molar mass, and (b) using the value for number of moles to determine the concentration of the solution (which for chemists can be expressed as molarity) using the formula molarity = moles/volume. The standard numerical solution is as follows:

$$\text{No. of moles of sodium nitrate} = \text{mass/molar mass}$$

$$= \frac{1.1}{23 + 14 + 16 \times 3} = 0.0129\,\text{mol}$$

$$\text{Molarity} = \text{number of moles/volume}$$

$$= \frac{0.0129}{0.25} = 0.052\,\text{M}$$

Subjects

Two Form 6 (Year 12) classes from one secondary school in Hong Kong were used for the study. All the students in these classes had completed a first course in volumetric analysis and had the same exposure to the topic. To obtain the experts and novices for the study, the students were first given a conventional paper-and-pencil problem-solving test on completing the topic. Students who made no procedural errors (i.e., neglecting arithmetic errors) and had a good conceptual understanding were classified as experts (for this topic) while those whose procedures were largely erroneous and had a poor conceptual understanding were classified as novices. The classification of experts and novices based on performance and identical exposure time contrasts to other studies in which the experts (often professors or graduate students) and novices (typically undergraduates) are of differing ages or experience levels and are selected without pre-testing. Both classes had been taught by the same teacher and although no instructions had been given as to how to teach the topic, it was noted that concepts and formulae were introduced in a way which emphasized definitions and mathematical formulae.

Procedure

Six expert and six novice students were selected from the classes at random. To ascertain their procedural and conceptual knowledge, two interview methods were employed. The first was the think-aloud procedure, in which students were instructed to talk out loud while solving problems or answering questions. The second procedure involved the use of probing questions after students had indicated that a task had been completed. All interviews were conducted in English in hour-long sessions held immediately after normal school hours. Although English is a second language for the students, it is used as the medium of instruction at the school, and no language-related difficulties were evident in either oral or written communication. Interviews were audio-taped and transcripts made to provide protocols of the sessions. From the protocols, inferences were made of students' procedural and conceptual knowledge. Whenever inferences could not fully account for students' responses, further questioning based on these inferences was carried out in later sessions and the inferences refined.

The methodology employed in the study has been widely used in information-processing psychology research and provides a tool for inferring problem-solving knowledge and mental processes (Van Someren Barnard and Sandberg 1994). Nevertheless, two points need to be noted. First, the use of just two classes precludes true random sampling so care is needed before generalizing the findings to the whole population of problem solvers for this topic. Second, because of the time consumed for interviewing in this type of research, only a small number of students are interviewed. However, generalization is still possible from small samples as Larkin and Rainard (1984) have shown.

Results and discussion

As the research yielded many protocols, reference is made in this article to just typical protocols in order to illustrate the findings of the study.

Procedural knowledge: problem-solving strategies

In solving the above problem, it is necessary to have a solution path leading from the information given in the problem statement to the goal. The strategies inferred for obtaining this path were means-ends analysis and two forms of working forward – one used by expert students in which a previously learnt procedure is recalled, and a more primitive version used by novice students when means-ends analysis fails.

Problem-solving strategies: expert students

The expert students in the study solved the problem rapidly using the standard two-step procedure described earlier. For most experts, the type of problem and its solution appeared to be immediately recognized. When asked if they recognized the problem, responses were similar to that given by one student:

> Yes, immediately! When I look at the question I think I know how to do it . . . Because it is common. We've done it many times before . . . I know how to do it.

After identifying the type of problem, 5 of the 6 expert students used a working forwards strategy and produced similar protocols. Consider the protocol for one expert student (ES1) shown in table 1. While talking aloud, the student does not refer to the goal statement but immediately starts executing the solution. He refers to the number of moles of sodium nitrate (lines 1 to 3) and calculates the numerical value followed by the calculation for the required molarity (lines 4 to 7).

The rapidity of the problem solving suggests that little if any search for a solution is necessary. A general procedure is probably already available in long-term memory which can be accessed and instantiated with data for the current problem. To use terms such as 'problem solving' and 'search strategy' may therefore be misleading. The problem has not been genuinely solved; the solution procedure already exists and does not have to be created. The expert student, while working forwards, is merely following a well-worn path and does not have to consciously make decisions as to the formula to be used at each step. This reinforces a suggestion made some time ago by Kramers-Pals, Lambrechts and Wolff (1983) that for an expert, a problem is no real problem at all but a standard problem for which a problem-solving sequence is applied almost automatically.

This problem-recognition-plus-working forwards strategy can be used whenever a problem is familiar and the solution known. However, for one of the 6 expert students (ES2), the solution did not seem to be immediately available. Part of her think-aloud protocol is given in table 1. She first identifies the goal of concentration, expressed as molarity (lines 1 to 2), and selects a correct formula linking this goal (molarity) with the data, viz., number of moles = molarity × volume (line 3). For this formula, data for volume but not moles is given, so she sets up a sub-goal to find numbers of moles (lines 5 to 6). This strategy suggests the use of means-ends analysis. With this strategy, the procedural steps are created in the reverse order to the procedure that is actually executed and written on paper, which, of course, is always forwards. Having created a solution path, she solved the problem quickly using the same numerical procedure as the other expert students. The use of means-ends analysis by this expert is in contrast

Table 1. Think-aloud protocols of two expert students for solving the basic problem in concentration.

Protocol of Expert Student 1 (ES1)

1. Calculate the number of moles of sodium nitrate ... Mass is 1.1 grams,
2. ... equals $1.1/(23 + 14 + 16 \times 3)$, equals ... 0.129.
3. [S uses calculator and writes while talking.]
4. Second, because molarity equals number of moles over volume, so
5. molarity equals 0.0129/0.25 equals ... 0.052 M.
6. [S again calculates and writes while talking.]
7. Therefore the concentration of the solution is 0.052 M.

(Note: ... signifies a pause.)

Part of the protocol for Expert Student 2 (ES2)

1. I've seen this [kind of problem] before ... What do I do first? ...
2. I'll first think of concentration ... it is the answer I must find. And then ...
3. I will use the equation: number of moles = molarity × volume, where molarity
4. is the concentration we are going to find.
5. To find out how many moles of sodium nitrate, I must know the atomic masses
6. so that I can calculate ... mole number of sodium nitrate.

to the findings of Larkin (1983), whose experts used working forwards exclusively. The different findings are probably a result of the way experts and novices are selected with the experts in this study being students and so having less experience than Larkin's experts. When student ES2 was asked to do a similar problem in a later session, she did use a working forward strategy and the switch from a means-ends strategy would be predicted as more experience is gained. This agrees with the findings of Sweller, Mawer and Ward (1983) mentioned earlier for the solving of physics problems. It is probable that the other expert students also used a means-ends analysis strategy when first faced with this type of problem, though as the topic had been taught before the present study was conducted, further research is needed to confirm whether this is indeed the case.

Problem-solving strategies: novice students

For the novice students in the study, the problem did not seem to have the familiarity or recognizable solution path as it did for the expert students. In contrast to the expert students, their performances were slower, they used a variety of formulae, some of which were erroneous, and the thinking aloud was characterized by frequent pauses and fewer comments. Follow-up questioning was therefore used in an attempt to elicit what students may have been thinking. Protocols of typical follow-up questioning are shown in table 2.

Student NS1 (table 2) thinks of the goal first and a formula to link this with given variables which again indicates the use of a means-ends analysis strategy. The student then went on to solve the problem correctly. But although the strategy used and the numerical procedure are very similar to those of expert student ES2 who also used a means-ends strategy, the novice student was less sure than the expert student of the correctness of the formulae used and commented that she was not sure if the answer was correct or not.

Other novice students appeared to means-ends analysis initially but could not think of a formula linking the goal to the data and appeared to switch strategies. The first few lines of the interview protocol of novice student NS2 in table 2 suggest such a switch. The subsequent order of processing shown in the protocol, from sodium nitrate (referred to from the problem statement) to number of moles of sodium nitrate to molarity (the goal), shows that a working forwards strategy is used. However, the student is struggling to come up with useful formulae that will enable a procedure to be generated. She did eventually manage to generate a two-step procedure for solving the problem using one correct formula and one erroneous formula as follows:

Step 1: Calculate the number of moles of sodium nitrate (correct using the formula: Moles = mass/molar mass).

Step 2: Calculate molarity (using an erroneous formula: molarity = moles × volume).

Comments by the student in table 2 also indicate that she, as with other novices, has little confidence in either the accuracy of the formulae used nor in the correctness of the numerical answer.

The working forwards strategy used by the novice student is a much more primitive version of that used by the expert students and is largely data driven.

Table 2. Protocols of follow-up interviews with novice students for solving the basic problem in concentration.

Protocol of a follow-up interview with Novice Student 1 (NS1)
Interviewer (I): What was the first thing you thought of after reading this problem?
Student (S): Moles equals MV [i.e., molarity × volume]
I: Why this first?
S: Because I want concentration. Concentration is molarity. The question gives volume and mass, so I think it may be true.
I: What did you think of next?
S: We have to get the number of moles. So we use the equation mass over molar mass [to get moles].

Protocol of a follow-up interview with Novice Student 2 (NS2)
I: What was the first thing you thought of after reading this problem?
S: The answer to find.
I: And then?
S: I have forgotten the formula for concentration.
I: What did you think of then?
S: Things in the question. Sodium nitrate. It is the main thing ... We can find the number of moles of sodium nitrate. Then we might be able to find other things.
I: Such as?
S: ... I'm not sure. Molarity, I hope!
I: Why molarity?
S: I think molarity is something like concentration.
I: Are you sure you can use number of moles later?
S: No. I hope I can.
I: What did you do next?
S: A formula using number of moles. One with molarity.
I: How sure are you that your answer is correct?
S: Not sure ... I think it is wrong.
I: Why?
S: The formula might be wrong. Molarity equals ... [volume × no. moles; S points to formula in written solution]. Is it correct?

Data from the problem statement is substituted directly into *any* formula that can be recalled. This process continues using given or derived data until a value for the goal variable is reached. If this is unsuccessful, as it was for some novices, the problem solving is terminated.

Most of the novice students in the study (those either interviewed or from the initial testing) used erroneous formulae in their written procedures for the problem. Examples of erroneous formulae used for determining molarity are:

$$\text{molarity} = \text{mass/molar mass}$$

$$\text{molarity} = \text{mass/volume}$$

$$\text{molarity} = \text{moles/molar mass}$$

$$\text{molarity} = \text{molar mass/volume}$$

$$\text{molarity} = \text{volume/moles}$$

$$\text{molarity} = \text{moles} \times \text{volume}$$

$$\text{molarity} = \text{mass} \times \text{volume}$$

A few erroneous formulae were also used in calculating the number of moles of solute. Later in this paper, when discussing students' conceptual understanding, causes are suggested for some of these erroneous formulae.

The switch by novices to a working forwards strategy was also observed by Larkin (1983) but only for the solving of 'harder' problems. The present findings show that this switch can also occur for 'easy' problems when less familiar. Taken together, the research shows that problem solvers, whether school students, undergraduates or university professors, have a variety of strategies available with the one actually employed depending on the familiarity of the problem and that switches are made to strategies which are more efficient or likely to lead to a solution.

Problem representation

All the students in the study set up an initial representation by referring to a key word such as 'molarity' or 'concentration', sometimes accompanied by concrete images of laboratory apparatus or procedures. The images appear to be an additional help for comprehending a problem perceived as more difficult or less familiar. One novice student was questioned on the use of images as follows:

> I: Do you think of an experiment with apparatus, or just formulae and data when starting this problem?
> S: An experiment ... I think of this ... [S draws a diagram of a solution in a beaker.]
> I: Does thinking of apparatus help you to solve the problem easier?
> S: No.

The use of key words and images of familiar objects with the chemistry problem is similar to the findings in physics (Chi *et al.* 1981, Larkin 1983, Slotta *et al.* 1995) though images seem to be less important in subsequent processing for the chemistry problem than for the physics problems.

Following the simple initial representation, the expert students set up two more representations of the problem. The first is a qualitative representation which contains a small number of key entities linked together in an outline procedure for solving the problem while the other is a mathematical representation corresponding to the numerical solution. Consider again the protocol in table 1 of the expert student (ES2) who used means-ends analysis. The key concepts of 'concentration', 'moles' and 'atomic masses' are identified and linked together to create a general qualitative procedure. Although formulae are referred to, this seems to be done in order to abstract the major entities needed for the procedure. The details are only worked out when the final mathematical solution to the problem is obtained. To check the accuracy of these inferences, the student was questioned about how the procedure was derived.

> I: Do you think about the details when first thinking of the solution?
> S: No! Just when I start the [mathematical] calculation I think about the details, for example, the mass, what are the atomic masses ... I first just find the main things.
> I: What are the main things in this problem?
> S: Just how many moles, molarity ... Volume [and] things like that are not important now.

The qualitative procedure derived by this student has the two steps (a) find the number of moles, and (b) find the molarity, corresponding to the standard outline of the solution procedure. Following the creation of this qualitative representation, the student used it to guide the use of formulae into which numerical data were substituted leading to a mathematical representation and numerical solution. Details, such as volumes and molar masses and how to calculate them which are not present in the parsimonious qualitative representation, were filled in at this stage.

The expert students who used a working forwards strategy also used a general procedure but, as discussed earlier under strategies, this is already available in long-term memory having been created at an earlier time (probably in much the same way as it was by student ES2 above). The protocol for student ES1 in table 1 is consistent with this hypothesis. As the student thinks aloud, he refers to key terms in the qualitative procedure (lines 1 and 4) while at the same time using it to generate the numerical solution to the problem.

Novice students, in contrast to the experts, have more difficulty forming a qualitative representation. Student NS1, like ES2, also used means-ends analysis but struggled while creating a procedure and was less certain of its correctness. From the protocol in table 2, she too refers to formulae but seems less able to abstract and link key variables, resulting in a qualitative representation that is not fully formed. Those novice students who switch from a means-ends analysis to the primitive working forwards strategy do not seem to form any qualitative representation at all. As the interview protocol in table 2 for novice student NS2 suggests, the student works forward blindly from one step to the next unable to link key variables into a coherent qualitative procedure. The procedure is formula driven, with a formula being selected and applied from one step to the next until a numerical answer for the goal term is arrived at.

In solving the chemistry problem, up to three representations are employed which is similar to the findings for the solving of physics problems (Larkin 1983). However, the present study finds that a qualitative representation can be set up when means-ends analysis is used (as it was for expert student ES2). In Larkin's (1983) study, means-ends analysis was only used by novices and was more formula driven as it was for the novices in this study. This difference is again probably a consequence of the way experts and novices are selected or categorised. Most likely, as the present findings suggest, there is a gradual shift in the representations and strategies used as expertise increases, from a primitive working forwards strategy without a qualitative representation of the problem to a means-ends analysis strategy with a partly or fully formed qualitative representation and finally to the working forwards strategy of an expert in which all three representations are present.

Conceptual knowledge and problem solving

The responses of all the expert students in the study, in both the written test and the interviews, suggest that their conceptual knowledge of the topic is congruent with standard chemistry knowledge. At the macroscopic level, they were able to show how the concentration of a solution varies in proportion to the amount of solute and in inverse proportion to the volume of the solution. When shown diagrams of two beakers with equal volumes of solution, students could readily state that the one with the greater amount of solute is more concentrated. Similarly, for equal amounts of solute, the solution with the smaller volume has the greater concentration. They were also able to manipulate amount of solute and volume simultaneously to compare the concentrations of two solutions. Furthermore, they appreciated that amount of solute and volume can be measured in any appropriate unit. For amount of solute, units mentioned by the expert students included mass, number of spoonfuls of solid, number of particles and number of moles of particles, the last being the standard chemical way of measuring amount of solute. Similarly, volumes could be in cm^3, litres or any appropriate unit. When amount was expressed as teaspoons of a substance and volume as litres, expert students had no difficulty calculating concentration in units of teaspoons per litre.

At the microscopic level, all the expert students held the standard particle model for concentration, that is:

$$\text{number of solute particles per unit volume} \propto \text{concentration.} \qquad (1)$$

Using this model, they were able to correctly complete diagrams such as those given for the questions in figure 1.

At the symbolic level, there is one key formula for concentration that must be understood, i.e. molarity = moles/volume. A satisfactory comprehension of this relationship can only be obtained through an understanding of a particle model for concentration and the concepts of the mole and molarity. All the expert students understood that molarity is a specific term for concentration and that the mole refers to 6×10^{23} particles of any substance.

Of more significance in this study is the kind of conceptual knowledge possessed by novices and how this is related to their procedural knowledge. At the macroscopic level, the novice students' understanding of concentration was similar to that of the expert students. They could compare concentrations of two solutions if amount of solute and volume of solution were manipulated separately. When both amount of solute and volume were varied together, they could usually get the correct answers, but unlike the expert students, only 2 of the 6 novices were able to explicitly use the idea of unit volume in explaining how the concentrations differed. Hence the relationship between concentration, amount of solute and volume was only implicit for most novice students.

When asked what measures could be used for amount of solute, all the novice students replied with 'mass'. Not one mentioned any everyday, non-chemical measures for either amount of solute or for volume of solution. Nor could any of the novice students suggest correct units for concentration when everyday measures such as spoonfuls of salt (for amount of solute) and litres (for volume) were given.

For novice students, the concept of concentration appears to be closely associated with that of density. Typical comments in this regard were:

> Concentration is the density of solute in a certain volume of water,

and:

> Concentration is the mass present in a unit volume.

In this second case, the student was not sure as to what the mass actually referred to. These associations appear to be related to students' perceptual experiences of dissolving solids as solutions become both denser and more concentrated as more solid dissolves. As one student commented when comparing the concentrations of two solutions having equal amounts of solute but different volumes:

> In this solution [larger volume], the solid is more spread out; the density is less than in the other solution [of lower volume].

At the microscopic level, only 2 of the 6 novice students held the correct particle model of concentration. For the other novices, the loose association of concentration, density and amount of solute together with a poor understanding of the role of unit volumes in concentration were also observed in the particle model. Figure 1 shows the responses of one novice student to interview questions used to assess this understanding. The erroneous model used was:

$$\text{number of particles of solute} \propto \text{concentration, irrespective of volume.} \quad (2)$$

With this relation, unit volumes are not considered. This misconception is shown in Question 2. Although the solutions have different volumes, the concentrations are the same so the student draws the same numbers of solute particles. Also, in the brief exchange following Question 3 (lines 12 to 13), the student reveals the erroneous association between concentration and amount of solute stating, incorrectly, that Solution B has three times as much solute as Solution A as it is three times more concentrated. The student is vaguely aware of the need to manipulate volume but finds this difficult. In Question 1 (lines 3 to 5), she briefly considers unit volume, and manages to answer correctly, but ignores it in the other two questions.

An interesting feature of the relationship between concentration and numbers of solute particles is the role of graphics. Unlike the diagrams in figure 1, if the amounts of solute being added are actually shown in a diagram, model 2 above is not revealed as the diagrams tend to facilitate mapping from the macroscopic level to the microscopic level. For example, when a diagram shows quantities of solute being added from spoons to beakers of water having equal or differing volumes, two novice students with erroneous model 2 correctly drew equal numbers of particles in each beaker. The inferred model for this is:

$$\text{number of particles of solute} \propto \text{amount of solute, irrespective of volume (amounts of solute shown in diagrams).} \quad (3)$$

While this model gives correct answers, it is still not model 1 above held by experts. Thus three different particle models of concentration were inferred for the novice students.

Another feature of the particle models is that, in contrast to the expert students, not one novice student ever talked about moles of solute particles. Therefore the dots shown in the diagrams in figure 1 are primitives and are not decomposable; one dot represents one particle of solute rather than one mole of particles.

The mole is a concept that has long caused difficulties with chemistry students. In studies with secondary school students, Novick and Menis (1976) detected the misconception that the mole is a certain mass rather than a certain number while many secondary students in a study by Schmidt (1984) regarded the mole as equivalent to molar concentration. And in a recent study of university freshmen and sophomores taking a chemistry course for scientists and engineers (Staver and Lumpe 1995) many errors were also observed, such as incorrectly defining the term including again defining the mole as a mass. Similar misconceptions were held by all the novices in the present study. Two inferred models for the number of moles of solute in a solution were:

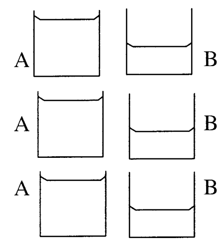
Question

In the diagrams, Solution A has twice the volume of Solution B. Complete the diagrams with dots representing solute particles to show that:

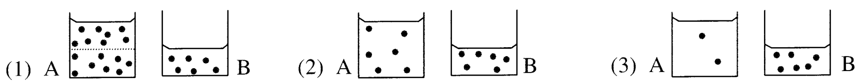
(1) Solution A is more concentrated than Solution B.

(2) the two solutions have the same concentration.

(3) Solution A is 1/3 the concentration of Solution B.



Response and protocol for one novice student



Question (1):
1. I think the particle density of A is closer than B. [S draws 6 dots in B.]
2. I think . . . more than 6 . . . about 12, greater than 12 [S eventually draws 16 dots in A].
3. I: Why did you draw a line across A?
4. S: I draw the line . . . the bottom part of A is equal to B. Then I think how many above
5. A of the line, then I think of the answer.

Question (2):
6. The question only gives me the concentration of A and B is equal. I think the density
7. of the particles in A is more spread out. The particles in B are closer.
8. [S draws 6 dots in B, then 6 in A.] The number of particles is equal.

Question (3):
9. [S draws 6 dots in B, 2 in A.] The question gives me A is one third the concentration of B.
10. And I also think the density . . . the space of the particles in A is very large . . . and in B
11. is very close.
12. I: Which solution has more solute?
13. S: B . . . it has a greater concentration than A. It has three times [the amount of solute as] A.

Figure 1. An interview question for a particle model of concentration and the response of a novice student.

1. Numbers of moles $\propto$ amount of solute
2. Numbers of moles $\propto$ molarity.

In the first model, amount is associated with mass. When comparing two solutions containing equal amounts of solute, one novice student commented:

> They [the two solutions] have equal masses of solute so I think they [number of moles] are the same.

With this model, correct inferences can be made regarding the relative numbers of moles of solute in a solution, even though the concept itself is poorly understood. As another student said:

> I only know moles is about the solute; I don't know what moles are.

However, as we will see below, this poor comprehension affects the formulae chosen when solving problems on concentration. The second erroneous model held by novice students relates the terms mole and molarity and appeared to be the result of confusion with the chemical symbol for molarity (mol) which is misinterpreted as mole.

The term molarity like the term mole, was not clearly understood. All the novice students knew that it was associated with concentration but were not sure of the connection. As one novice student commented:

> Molarity is something to do with concentration. I've heard the teacher say it many times.

And, as with the term concentration, molarity and density are confused. Typical comments were:

> Molarity is the mass of solute divided by the volume of solution,

and

> Molarity is the mass of substance in a certain [volume of] liquid.

To calculate numerical values for concentration, just one formula is needed, viz., molarity = moles/volume. Novice students used a variety of formulae which were poorly related to chemical concepts. Some of the novice students used the formula:

Concentration (Molarity) = mass of solute/volume

which is the result of the strong association of concentration with density mentioned earlier. This formula was used for problems which asked for the calculation of concentration as well as those asking for molarity. Other formulae for calculating molarity appeared to result from a hazy knowledge of the correct formula. For example, one novice student used the formula: moles = molarity/volume. This formula contains the same terms as the correct formula but with an erroneous relationship between the terms. The formula is not merely an incorrect transposition, as the student always wrote it down in the way shown before transposing it. One of the novice students, though having a poor conceptual understanding of concentration, was the strongest procedurally, as she had memorized the correct formulae and how to apply them. This confirms findings in other studies (e.g. Nurrenberg and Pickering 1987, Stewart 1985), that students can be successful at solving problems in a topic but be unable to link their procedural knowledge to the conceptual framework of that topic.

The other formula needed to solve the problem is that for calculating numbers of moles, i.e moles = mass/molar mass. This was less of a problem for the novice students who seemed to be familiar with it from earlier studies even though, as mentioned above, they did not have a clear conception of the mole.

Summary and implications for practice

This study reports on the solving of a basic quantitative problem in volumetric analysis by expert and novice secondary school students. It examined the kinds of strategies used, how the problem was represented together with the conceptual knowledge underlying the problem solving. Similarities and differences were found with problem solving in physics. Both expert and novice students set up an initial representation of the problem by identifying key words from the problem statement. Images of laboratory apparatus are also sometimes formed, though this connection with familiar objects seems to be less important for subsequent processing than it is for physics problems. Most experts, being familiar with the type of problem, would access a general qualitative procedure for solving the problem from memory. This procedure is parsimonious, consisting of a small number of key variables linked together. Employing a working forwards strategy, the expert students use this qualitative procedure to guide the selection of appropriate formulae to construct a mathematical representation of the problem and a numerical solution. Novice students, in contrast, after identifying key words, employ a means-ends analysis strategy in the first instance. However, this often fails as students cannot think of a formula linking the goal with given data. In this case, they switch to a simple working forwards strategy in an attempt to generate a numerical solution using any available formulae into which given or derived data are substituted. The strategies are formula driven and novices either omit the quantitative procedure or only partially generate one.

The conceptual knowledge of the expert students in the study was accurate and was fully integrated with the two formulae needed to solve the problem. Novices, however, had misconceptions or a hazy understanding for the key concepts of molarity and the mole. This resulted in a variety of erroneous formulae such as molarity = mass/volume which is due to the misconception that concentration and density are equivalent. Sometimes formulae were memorized and applied algorithmically.

Instructional techniques to assist practitioners and to help novices become more like experts are derived by comparing the processing and knowledge involved in expert and novice problem solving. Three areas where the differences between the expert and novice students are significant are in (a) conceptual understanding, (b) the problem-solving strategies used and (c) the use of a qualitative representation. Based on these areas, a number of recommendations are given below to guide teaching practice.

1. *Teach for conceptual understanding*

Many of the concepts in volumetric analysis are abstract in nature and cause difficulties for students. Consider molarity. This concept is often poorly understood and not linked to the formula: molarity = moles/volume. It should help if the concept is introduced concretely. Molarity is associated with concentration which can be introduced concretely by relating it to *coloured* substances such as concentrated and dilute orange juice. To show the relationship between concentration, amount of solute and volume, coloured solutions can again be used. For example, by adding 1 spoonful of orange potassium dichromate crystals to a unit volume of water (say half a beaker full) and 2 spoonfuls to another equal volume, students will see from the intensity of the colour how amount of solute affects concentration. Similarly, by keeping the number of spoonfuls of solid constant and varying the volume, students can better comprehend how concentration is affected by volume. This leads more easily into concentration as amount of solute per unit volume of solution for which the chemical terms of molarity (for concentration) and moles (for amount) can then be introduced. In this way the concept of molarity is more likely to be understood and linked with the formula molarity = moles/volume.

2. *Encourage identification of key words and qualitative thinking*

The use of key words forms a significant part of the initial problem representation which for experts, leads to a qualitative procedure of a problem. To encourage identification of key words and qualitative thinking, teachers could allow students the opportunity to *talk aloud* while solving a problem. This would include talking about the key words in problems and the derivation of *qualitative, non-mathematical* procedures for problems and could be carried out either at the chalkboard or by getting students to work together, say in pairs, with instructions to derive general procedures rather than mathematical solutions. Talking aloud would also help teachers and students to identify misconceptions or reveal areas of knowledge not clearly understood but which are needed to solve problems.

3. *Consider strategies used by students*

Teachers, being experts and familiar with a problem, may tend to use a working forwards approach when demonstrating how to solve a problem. But for students, when a problem is unfamiliar, mean-ends analysis is the strategy employed initially. During talk-aloud sessions, teachers could therefore help students to focus on the goal of a problem then set up appropriate sub-goals in order to create a qualitative procedure. Initial problem solving will be slow but as the solution path becomes familiar, students should switch to the more efficient working forwards strategy used by experts.

4. *Practice*

Once students have derived *and* understood procedures for problems, they should be given plenty of practice to master the procedures and to encode them in long-term memory. Too often, we as teachers move on to new topics before slower students get sufficient practice to do this. Once encoded, procedures can be readily accessed from long-term memory and used almost automatically as is done by experts. This could be beneficial when longer, more complex problems are solved which contain basic procedures as components of the overall solution.

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